University of Ljubljana
Institute of Mathematics, Physics and Mechanics
Department of Mathematics
Jadranska 19, 1000 Ljubljana, Slovenia

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THE PETERSEN GRAPH IS NOT 3-EDGE-COLORABLE – A COUNTING ARGUMENT PROOF

Reza Naserasr   Riste Škrekovski

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The Petersen graph is not 3-edge-colorable - a counting argument proof

Reza Naserasr[a*] Riste Škrekovski[b,c†]

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[a] Department of Mathematics, Simon Fraser University, Burnaby, B.C. V5A 1S6, Canada, rnaseras@math.sfu.ca

[b] Department of Mathematics, University of Ljubljana, Jadranska 19, 1111 Ljubljana, Slovenia, skreko@fmf.uni-lj.si

[c] Pacific institute for the Mathematical Science (PIMS), Simon Fraser University, Burnaby B.C. V5A 1S6, Canada

Abstract

It is well know that the Petersen graph is not 3-edge-colorable and most proofs for it use a case by case argument or use very symmetric structures of the Petersen graph. Here we give an alternative proof, by a counting argument, but without much use of the symmetries of the Petersen graph.

J. Petersen introduced the most well known graph, the Petersen graph, as an example of a cubic bridgeless graph which is not Tait colorable, i.e. it is not 3-edge-colorable. It is easy to see the equivalence between the following statements but most proofs for each of them use a case by case argument or use the very symmetric structure of the Petersen graph [1].

Theorem 1 *For the Petersen graph P the followings are equivalent:*

(1) P is not 3-edge-colorable;

(2) P is not Hamiltonian;

(3) P does not admit a nowhere-zero 4-flow.

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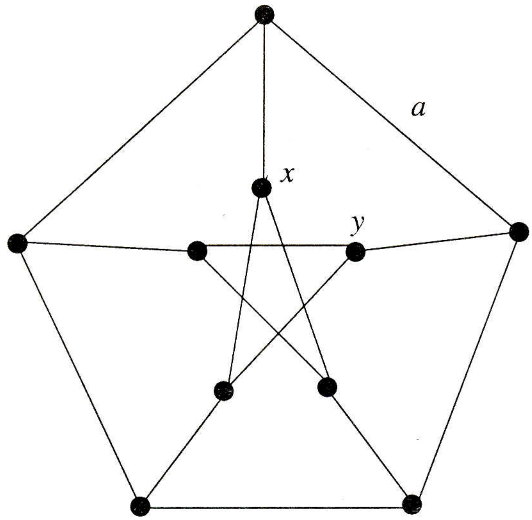

Figure 1: The Petersen graph

Here we give an alternative proof, by a counting argument, but without much use of the symmetries of the Petersen graph.

Proposition 1 *The Petersen graph is not 3-edge-colorable.*

Proof: Suppose it has a 3-edge-coloring and consider the Petersen graph of Figure 1, with the edge colored a. Then there have to be two (distinct) edges from the inner cycle colored a, one incident with vertex x and the other incident with vertex y. In other word, every color appearing on the outer cycle is appearing on at least two distinct edges in the inner cycle. But all three colors must appear on the outer cycle and $5 < 6$.

■

References

[1] D. A. Holton and J. Sheehan, *The Petersen Graph*, Cambridge University Press, Cambridge (1993).