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A CLASSIFICATION OF CUBIC BICIRCULANTS

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A Classification of Cubic Bicirculants

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Abstract

The well-known Petersen $G(5,2)$ admits a semi-regular automorphism α acting on the vertex set with two orbits of equal size. This makes it a *bicirculant*. It is shown that trivalent bicirculants fall into four classes. Some basic properties of trivalent bicirculants are explored and the connection to combinatorial and geometric configurations are studied. Some analogues of the polycirculant conjecture are mentioned.

1 Introduction and Classification

The object of this study are trivalent (cubic) graphs admitting an action of a cyclic group having two equally sized vertex orbits. Such graphs are called *bicirculants*. The automorphism α that generates the corresponding cyclic group is said to be *semi-regular*. Let G be a bicirculant and let α be the corresponding semi-regular automorphism. By $V(G) = V_1 \cup V_2$ we denote the decomposition of the vertex set into the two orbits where

$$V_1 = \{u_0, u_1, ..., u_{n-1}\}$$
$$V_2 = \{v_0, v_1, ..., v_{n-1}\}$$

and $\alpha(u_i) = u_{i+1}, \alpha(v_i) = v_{i+1}, i = 0, 1, ..., n-1$. Note that all computations are $\mod n$.

There are two *types* of edges with respect to α. An edge e that has both of its endpoints in the same orbit is said to be a *rim edge* (R), otherwise, if the two endpoints belong to distinct orbits, the edge is said to be a *spoke* (S). Now we distinguish four classes of bicirculants (with respect to a given semi-regular element α). Consider the types of edges incident with a common vertex. If all the edges are rim edges, we denote the corresponding class by $3R$, if two of them are rim edges the class is denoted by $2R + S$, if only one edge is a rim edge, the class is denoted by $R + 2S$, while the remaining class, composed of only spoke edges is denoted by $3S$.

Recall that the *Möbius ladder* M_n on $2n$ vertices is a graph on the vertex set $\{v_0, v_1, ..., v_{2n-1}\}$ with two types of edges $v_k \sim v_{k+1}, v_k \sim v_{k+n}, k = 0, 1, ..., n-1$.

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Also, a *generalized Petersen graph* $G(n, r)$ is a graph on the vertex set

$$\{u_0, u_1, ..., u_{n-1}, v_0, v_1, ..., v_{n-1}\}$$

with the adjacencies:

$$u_k \sim u_{k+1}, v_i \sim v_{k+r}, u_k \sim v_k, k = 0, 1, ..., n-1.$$

In particular, $G(5, 2)$ is the Petersen graph, $G(10, 2)$ is the dodecahedron graph, and $G(n, 1)$ is also known as the $n-$*prism* Π_n.

A *circulant graph* is a possibly disconnected Cayley graph of a cyclic group. It is well-known that each circulant $C(n, S)$ can be described by two parameters: an integer n and s *symbol* $S \subset \mathbb{Z}_n$, such that $0 \notin S, S = -S$. It follows, that for a cubic circulant we have $S = \{i, -i, n/2\}$ and therefore n has to be even. Each cubic circulant graph either consists of isomorphic copies of $r-$prisms Π_r or Möbius ladders.

Bicirculants have been studied in the past [18]. They form a subclass of important class of graphs, called polycirculants; see for instance [17, 19, 18] and also [2, 23].

We are interested only in simple graphs (graphs with no loops or parallel edges. However in describing them we use general graphs and even pregraphs, allowing parallel edges, loops and even half-edges.

Proposition 1. *In any cubic bicirculant graph the type of vertex is constant; i.e. all the vertices have the same type.*

Proof. Since the type of a vertex is preserved by the automorphism α the vertices in the same orbit have the same type. As both vertex orbits have the same size, the number of spokes per vertex is constant. □

As we mentioned earlier, this decomposition in general may depend on α. We shall explain the structure of each of the four classes.

Class $3R$. Since there are no edges between V_1 and V_2 the graph $G|V_i$ induced on $V_i, i = 1, 2$, is a union of connected components. $G|V_i$ is therefore a cubic circulant graph. This means that $n = |V_1| = |V_2|$ has to be an even number. The only connected cubic circulants are Möbius ladders and odd prisms. This completely reveals the structure of graphs in $3R$. None of them is connected. Each graph from $3R$ has a number of vertices divisible by 4. Girth is at most 4. The graphs on $2n$ vertices can be described by three parameters n, i, j and are denoted by $T(n, i, j)$ where n is an even integer; see Figure 1. Since $T(n, i, j), T(n, j, i)$ and $T(n, n-i, n-j)$ are isomorphic, we may assume $0 < i \le j \le n-1, i+j \le n$. There are three types of edges:

$$u_k \sim u_{k+i}, k = 0, 1, ..., n-1.$$

$$v_k \sim v_{k+j}, k = 0, 1, ..., n-1.$$

$$u_k \sim u_{k+n/2}, v_k \sim v_{k+n/2}, k = 0, 1, ..., n-1.$$

In order to describe the structure of $T(n, i, j)$ we need new notation. let $g(n, k) = \gcd(n, k)$, if $n/gcd(n, k)$ is even and let $g(n, k) = \gcd(n, k)/2$ if $n/gcd(n, k)$ is odd. Furthermore, let X_t denote a Möbius ladder $M_{t/2}$ if t is even or a prism Π_t, if t is odd. Let $r = n/\gcd(n, i)$ and let $s = n/\gcd(n, j)$.

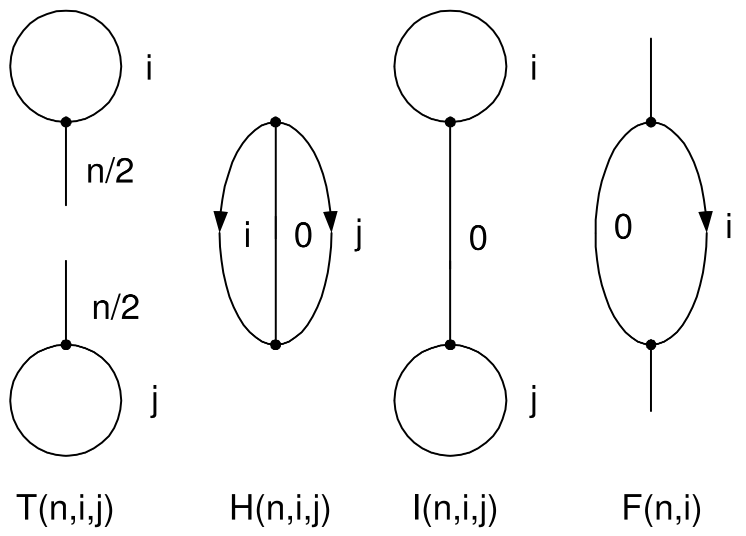

Figure 1: Each cubic bicirculant on $2n$ vertices is a cyclic cover over one of the four cubic pregraphs on two vertices. The voltage group in each case is $\mathbb{Z}_n$.

Hence $T(n, i, j)$ consists of $g(n, i)$ copies of X_r and $g(n, j)$ copies of X_s. Clearly each prism or Möbius ladder lies entirely in one of the two orbits. For instance $T(6, 1, 2)$ has one component isomorphic to Π_3 and the other to M_3; see Figure 2.

Class $3S$. All edges have one endpoint in V_1 and the other one in V_2. The graphs therefore coincide with cubic cyclic Haar graphs introduced in [16]. They can be described by three parameters n, i, j and are denoted by $H(n, i, j)$. We may assume $0 < i < j \leq n-1, i+j \leq n$.

$$u_k \sim v_k, k = 0, 1, ..., n-1.$$
$$u_k \sim v_{k+i}, k = 0, 1, ..., n-1.$$
$$u_k \sim v_{k+j}, k = 0, 1, ..., n-1.$$

Class $2R + S$. Each vertex from V_1 is incident with two rim edges and one spoke. The same is true for each vertex from V_2. The class of graphs coincides with the so-called I-graphs and can be described by three parameters n, i, j and are denoted by $I(n, i, j)$. These graphs have been studied in [3]. Using the same argument as above we may assume $0 < i \leq j \leq n-1, i+j \leq n, i \neq n/2, j \neq n/2$. Note that $i = n/2$ and $j = n/2$ are forbidden as they would result in parallel edges.

$$u_k \sim v_k, k = 0, 1, ..., n-1.$$
$$u_k \sim u_{k+i}, k = 0, 1, ..., n-1.$$
$$v_k \sim v_{k+j}, k = 0, 1, ..., n-1.$$

Class $R + 2S$. Each vertex from V_1 is incident with a single rim edge and two spokes. The same is true for each vertex from V_2. The class of graphs $F(n, i)$ can be described by two parameters n and i with $0 < i \leq n/2$ and n even.

$$u_k \sim v_k, k = 0, 1, ..., n-1.$$
$$u_k \sim v_{k+i}, k = 0, 1, ..., n-1.$$
$$u_k \sim u_{k+n/2}, v_k \sim v_{k+n/2}, k = 0, 1, ..., n-1.$$

It is not hard to see that $F(n,i)$ are composed of disjoint Möbius ladders or prisms. Note that a different choice of α would put this graph in the class $3R$.

Let us summarize the previous discussion in a form of a theorem. We provide a short proof that uses voltage graphs and covering graphs; for definitions and similar use of covering graph technique see for instance [2, 22]. The base spaces are pre-graphs. These are graphs with half-edges. The only voltages that can be assigned to half-edges are involutions. If we want to have covering spaces to be graphs (without half-edges), the voltage has to be a fixed-point free involution. Since the voltage group in our case is $\mathbb{Z}_n$, the only admissible voltage on a half-edge is $n/2$. When a base graph contains a vertex with incident pair of half-edges they can be replaced by a loop with a voltage $n/2$. This reduces the number of possible base pre-graphs and hence the number of cases to four. In the drawing of voltage graph G with voltage assignment $\gamma : E(G) \to \mathbb{Z}_n$ we omit the arrow on an edge e if it can be reversed without changing the graph, i.e. if $\gamma(e) = -\gamma(e)$.

Theorem 2. *Any cubic bicirculant on $2n$ vertices is isomorphic to one of the following graphs: $T(n,i,j), H(n,i,j), I(n,i,j)$ and $F(n,j)$.*

Proof. By definition, a cubic bicirculant is a cyclic covering graph over a cubic pregraph on two vertices. Since there are exactly four cubic pregraphs on two vertices, the bicirculants naturally fall into four classes as shown in Figure 1. In each case we have, in addition to n, the number of layers of each fiber, two parameters that arise by voltage assignment to the edges; compare [22]. □

Figure 2 depicts some graphs described in Theorem 2. In the remainder of the paper we use the results that were obtained during the study of graphs $H(n,i,j)$; [16] and $I(n,i,j)$; [3]. According to [9] a *zero-symmetric graph* is a vertex-transitive, trivalent graph whose edges are partitioned into three orbits by its automorphism group.

2 Some Basic Properties

Here we will explore three basic properties of bicirculants.

Proposition 3. *A cubic bicirculant graph G is connected if and only if:*

1. $G = H(n,i,j)$ *and* $\gcd(n,i,j) = 1$.
2. $G = I(n,i,j)$ *and* $\gcd(n,i,j) = 1$.
3. $G = F(n,i)$ *and* $\gcd(n,i) = 1$ *or* $\gcd(n,i) = 2$ *and* $n/2$ *is odd.*

Proof. Since none of the graphs $T(n,i,j)$ is connected we have to consider only the remaining three classes. Case 1 follows from [16], Proposition 3.1. Case 2 is described in [3]. The remaining case $F(n,i)$ is clearly disconnected if $\gcd(n,i) > 2$ and is connected if $\gcd(n,i) = 1$. If $\gcd(n,i) = 2$ the spokes alone form two cycles that are connected by the rim edges if and only if both cycles are odd. □

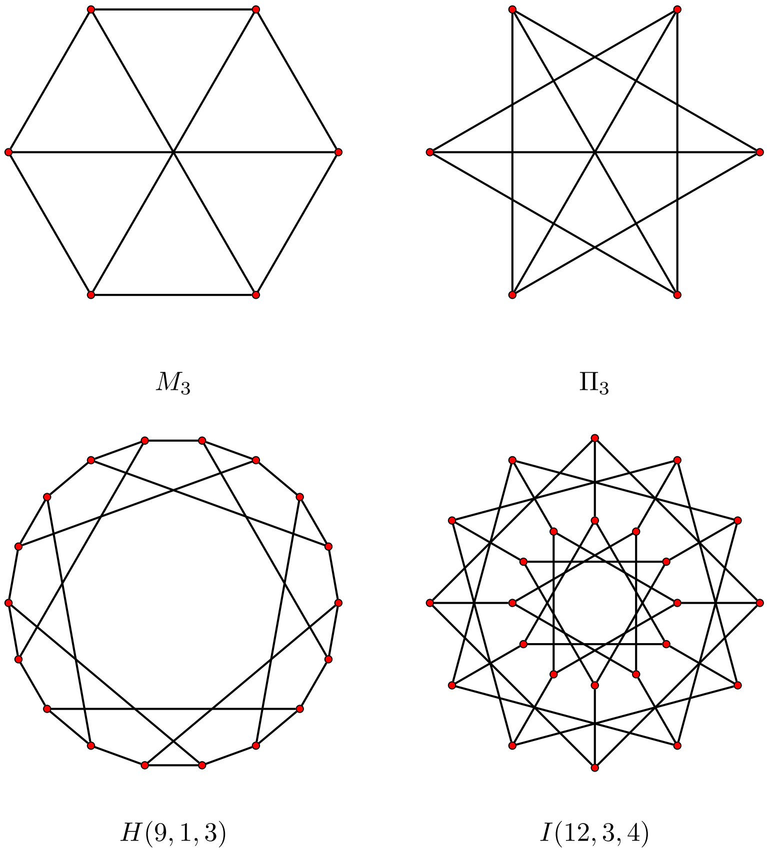

Figure 2: $T(6,1,2)$ has two components M_3 and Π_3 shown above. $H(9,1,3)$ is the smallest zero-symmetric graph; see [9]. $I(12,3,4)$ is the smallest $I-$graph that is not isomorphic to a generalized Petersen graph. The family of graphs $F(n,i)$ has no new members.

Theorem 4. *Each cubic bicirculant connected graph G is 3-connected.*

Proof. In each of the three classes of graphs one can find three vertex disjoint paths between any pair of vertices. $\square$

Since the girth of any Möbius ladder or prism graph is at most 4, it follows that $\text{girth}(T(n,i,j)) \leq 4$ and $\text{girth}(F(n,i)) \leq 4$. In [16] it was shown that $\text{girth}(H(n,i,j)) \leq 6$. and in [3] it was shown that $\text{girth}(I(n,i,j) \leq 8$. In these references one can find recipes for computing girth of these graphs.

Since the semi-regular automorphism α produces only two orbits, there are only two cases concerning vertex transitivity.

Proposition 5. *A cubic bicirculant graph G is vertex transitive if and only if*

1. $G = T(n,i,j)$ *and* $\gcd(n,i) = \gcd(n,j)$
2. *any* $G = H(n,i,j)$

3. $G = I(n,i,j)$ *and* $\gcd(n,i) = \gcd(n,j) = 1$, *$G$ is isomorphic to the dodecahedron graph $G(10,2)$ or else there exists and integer r such that* $j = r.i \pmod n$ *and* $i = \pm r.j \pmod n$.

4. *any* $F(n,i)$.

Proof. The graphs in the first case are vertex-transitive if and only if they have all connected components isomorphic. The second case is covered in [16]. Each cyclic Haar graph is a Cayley graph. The third case follows from [3] where it is shown that the only transitive I-graphs are certain generalized Petersen graphs. The class of vertex-transitive generalized Petersen graphs was established in [10]. The last case has all connected components isomorphic and each of them is vertex-transitive.

□

Proposition 6. *A cubic bicriculant graph G is bipartite if and only if*

- *any* $G = T(n,i,j)$, *n even, and* $n/\gcd(n,i)$ *and* $n/\gcd(n,j)$ *are odd numbers.*
- $G = H(n,i,j)$
- $G = I(n,i,j)$ *and n is even and i and j are odd.*
- $G = F(n,i)$, *n is even and i is odd.*

Proof. In the first and the last case we only have to distinguish even Möbius ladders from odd prisms. The second case follows from [16] and the third one from [3]. □

Proposition 7. *A cubic bicirculant graph G is a Cayley graph if and only if*

- $G = T(n,i,j)$ *and* $\gcd(n,i) = \gcd(n,j)$
- *any* $G = H(n,i,j)$.
- $G = I(n,i,j)$ *and* $\gcd(n,i) = \gcd(n,j) = 1$ *and there exists and integer r such that* $j = r.i(\mod n)$ *and* $i = r.j(\mod n)$.
- *any* $G = F(n,i)$.

Proof. In the first case we have to make sure that all connected components are isomorphic. The second case follows from [16] and the third one from [3]. The last case is trivial. □

3 Bicirculants and Configurations

Finally, let us touch on a problem of configurations. A combinatorial (v_3) configuration is an incidence structure composed of v points and v lines and there are exactly three points on each line and there are exactly three lines passing through each point. Furthermore, any pair of distinct points determines at most one line. For the basic facts on configurations, the reader may consult for instance [2]. These configurations were counted in [1] for $v \leq 18$. There is an important connection between configurations and graphs. A *Levi graph*

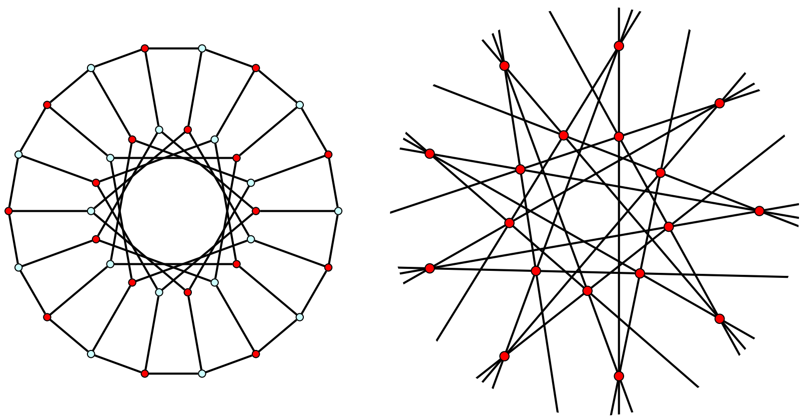

Figure 3: The graph $G(18,5) = I(18,1,5)$ is the smallest bicirculant graph that is a Levi graph of a triangle-free configuration; compare [4]
.

(see [8]) of a (v_3) configuration is a bipartite cubic graph where black vertices represent points and white vertices represent lines of the configuration. The incidence of point and line is represented as an edge of the corresponding Levi graph. The girth of any Levi graph is at least 6. A cycle of length 6 in the Levi graph corresponds to a triangle in the combinatorial configuration. Similarly, a cycle of length 8 corresponds to quadrangles in the configuration. In [1] triangle-free configurations have been counted as well. This means that cubic bipartite graphs of girth at least 8 have been counted. Recent paper [4] studies small triangle-free configurations.

If we restrict our attention to the configurations whose Levi graphs are bicirculants, then each configuration contains either a triangle or a quadrangle. There is another approach to this connection. In [2] polycyclic configurations have been studied. Levi graphs of polycyclic configurations are polycirculants. A bicirculant is a polycirculant of order 2. The polycirculants arising from polycyclic configurations have an additional property that each orbit is an independent set of vertices. Let us call such polycirculants *independent*. The only independent bicirculants are cyclic Haar graphs. They correspond to cyclic configurations.

It is possible to characterize Levi graphs among the bicirculants.

Theorem 8. *A cubic bicirculant is a Levi graph of a combinatorial configuration if and only if*

- $G = H(n,i,j)$ *and* $0 \neq i, 0 \neq j, i \neq j, 2i \neq j, 2j \neq i, i+j \neq 0, n \neq 2i, n \neq 2j$.
- $G = I(n,i,j)$ *and* n *is even,* i *and* j *are odd and* $n \neq 4i, n \neq 4j, i \neq j, i+j \neq 0$.

Proof. Only the second and third case apply for girth to be at least 6. The claim follows from [3], Theorems 3 and 7. For bipartite $I(n,i,j)$ the conditions for the girth can be simplified since no odd cycles are possible. □

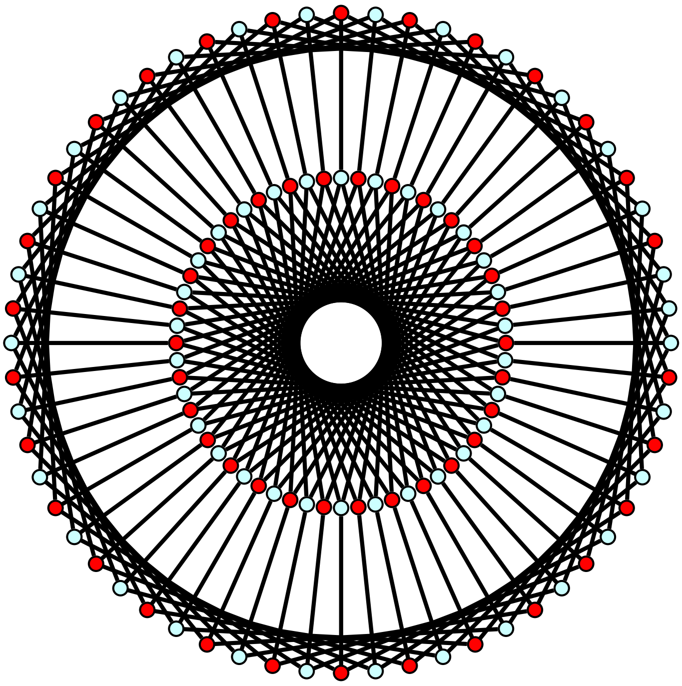

Figure 4: In the Table 4 only one graph from each equivalence class is presented. For instance there are eight graphs isomorphic to the smallest one $I(60,5,3)$. One of the isomorphs is $I(60,25,9)$ and is easier to visualize. The paper [3] addresses the problem of isomorphism of I-graphs in detail.

Theorem 9. *A cubic bicirculant is a Levi graph of a combinatorial triangle free configuration if and only if* $G = I(n,i,j)$ *and* n *is even,* i *and* j *are odd and* $0 \neq 2i, 0 \neq 2j, 0 \neq 4i, 0 \neq 4j, i \neq j, i+j \neq 0, 3i \neq j, 3j \neq i, 3i \neq -j, 3j \neq -i, 0 \neq 6i, 0 \neq 6j, 2i \neq 2j, 2i+2j \neq 0, 3i \neq j, 3j \neq i, 3i \neq -j, 3j \neq -i.$

Proof. Only certain I-graphs have girth greater than 6. The result follows from [3], Theorems 3 and 7. □

If we want the graph to be different from any generalized Petersen graph, we have to add two more conditions: $\gcd(n,i) > 1, \gcd(n,j) > 1$.

Using the above theorems we produced our tables.

In addition to combinatorial configuration we also have geometric configurations of points and lines in the Euclidean plane. The problem is which combinatorial configurations admit geometric realizations.

According to the Theorem 4 all connected configurations of this paper are 3-connected. This means that the corresponding Levi graph is 3-connected. The smallest two cyclic configurations (7_3) and (8_3) are not geometrically realizable.

7	3	1	8	3	1	9	3	1	10	3	1	11	3	1
12	3	1	12	8	1	12	9	8	13	3	1	13	10	1

Table 1: Parameters for the graphs $H(n,i,j)$ of girth 6. They constitute the class of Levi graphs of cyclic (n_3) configurations $n \leq 13$; see [1].

8	3	1	10	3	1	12	5	1	14	3	1	16	3	1
16	5	3	18	3	1	20	3	1	20	7	3			

Table 2: Parameters for the bipartite graphs $I(n,i,j)$ of girth at least 6, $n \leq 20$.

18	5	1	22	5	1	24	5	1	24	5	3	24	7	1
26	5	1	26	5	3	28	5	1	30	7	1	30	7	3
30	11	1												

Table 3: Parameters for the bipartite graphs $I(n,i,j)$ of girth 8, $n \leq 30$. For the smallest one see Figure 3

60	5	3	70	7	5	84	7	3	90	5	3	90	9	5
110	11	5	120	5	3	120	9	5	126	7	3	126	9	7
130	13	5	132	11	3	140	7	5	150	5	3	150	9	5
154	11	7	156	13	3	168	7	3	168	9	7	170	17	5
180	5	3	180	9	5	180	21	5	182	13	7	190	19	5
198	11	3	198	11	9									

Table 4: Parameters for the bipartite graphs $I(n,i,j)$ that are not generalized Petersen graphs of girth 8, $n \leq 198$.

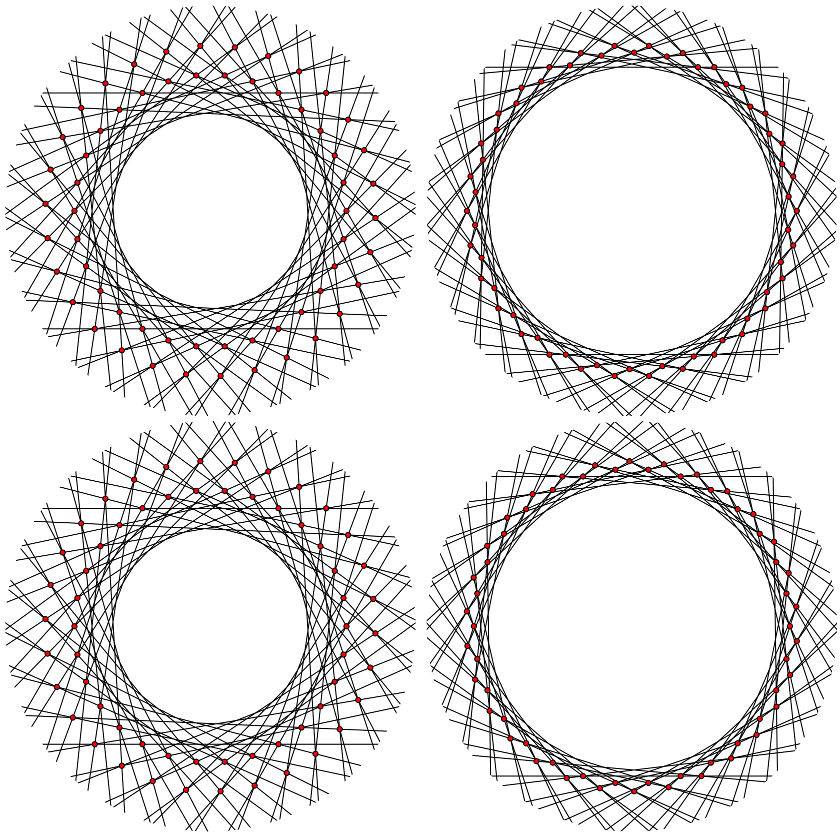

Figure 5: Using the theory from [3] we can construct 4 geometrically distinct (60_3) triangle free astral configurations with the same Levi graph $I(60,5,3)$.

In [4] the following conjecture is posed:

Conjecture 1. *Every 3-connected combinatorial* (n_3) *configuration with* $v > 10$ *is geometrically realizable.*

If this Conjecture is true, then all other configurations of this paper are geometrically realizable. Note that for $n \leq 12$ there are only three examples that violate this Conjecture. In addition to the Fano plane (7_3) and the Möbius-Kantor configuration (8_3) there is only one more. Among the ten (10_3) combinatorial configurations, there is one that admits no geometric realizations.

The smallest I-graph of girth 8 is the generalized Petersen graph $G(18,5)$ that has an important role in connection with astral configurations, see [4]. The smallest non-generalized Petersen I-graph of girth 8 is $I(60,5,3)$, see Figure 4; compare [3]. There are eight $I-$graphs isomorphic to $I(60,5,3)$. However, there is only one self-dual combinatorial configuration resulting from these graphs. As we have seen in Figure 6 this combinatorial (60_3) configuration admits a geometric realization. Using theory developed in [2] we can construct other symmetric geometric realizations of this combinatorial configuration. For instance, exactly four of them are astral; see Figure 5.

This rises one more question that would be interesting to address. We have

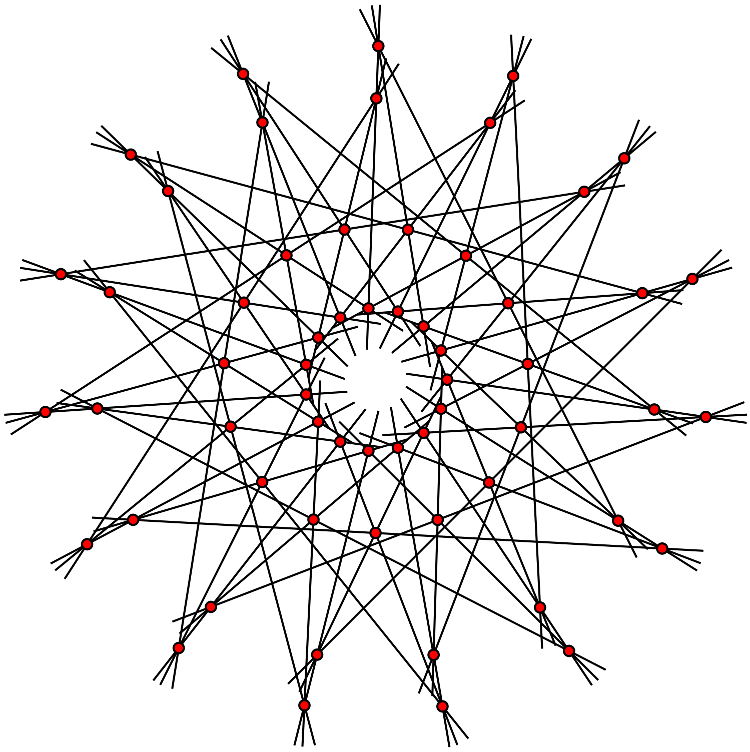

Figure 6: The graph $I(60, 25, 9)$ from Figure 4 is a Levi graph for a number of geometric configurations. Four astral ones are shown in Figure 5. Some features are more clearly visible in this stellar geometric (60_3) configuration.

seen that different I-graphs can be isomorphic. On the other hand the same combinatorial configuration may produce distinct geometric configurations. A natural question is: how many distinct geometric forms can a combinatorial configuration possess? Some geometric configurations admit a lot of freedom. Sometimes we may continuously move a point and keep the required incidences until we reach a different configuration. Obviously, there are only finite number of possibilities. It would be interesting to explore this idea and at least give some non-trivial upper bound for a number of distinct geometric realizations of given (v_3) configuration. Obviously, before we can answer this question we have to define when two geometric configurations are "the same". Since any geometric configuration is a line arrangement [24] we may have to use the idea of equality of line arrangements. But that is a subject for a separate work. We conclude this topic by exhibiting a projectively rigid configuration (4_2) Figure 7 that admits six Euclidean configurations and defines four Euclidean line arrangements.

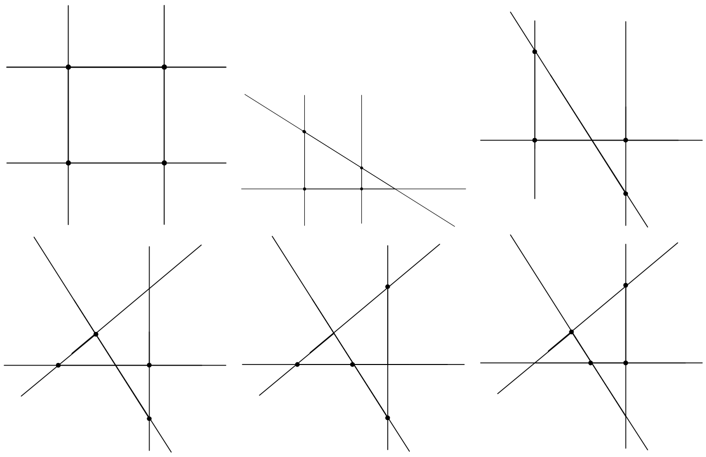

Figure 7: Six Euclidean realizations of the combinatorial (4_2) configuration. Projectively they are all indistinguishable. Note that the 6 Euclidean realizations define only 4 distinct Euclidean line arrangements.

4 Bicirculants and the Polycirculant Conjecture

The original polycirculant conjecture as stated in 1981 by Dragan Marušič [17] can be formulated as follows:

Conjecture 2. *Every vertex transitive graph admits a non-trivial semi-regular automorphism.*

In [21] the authors have proven the conjecture for the case of cubic graphs.

Motivated by configurations one can try to modify the conjecture to a different environment.

Conjecture 3. *Every point- and line- transitive combinatorial configuration is a polycyclic configuration.*

If we translate this conjecture to the language of graphs and we forget about the girth 6, we can reformulate the conjecture as follows:

Conjecture 4. *Every bipartite graph that has all black vertices in a single orbit and all white vertices in a single orbit, admits a semi-regular automorphism.*

When Branko Grünbaum introduced the notion of an *astral* configuration in [14] he had in mind geometric configurations. A geometric (v_3) configuration is called *astral* if its group of isometric symmetries has only two point orbits and two line orbits. The only groups of symmetry possible for an astral configuration are cyclic groups or dihedral groups, see Figure 8.

This gives rise to the following generalization of a semi-regular element $\alpha \in \operatorname{Aut} G$. Instead of semi-regular element α one can view the corresponding semi-regular cyclic group, generated by α. More generally, a subgroup $\Gamma \leq \operatorname{Aut} G$ acting on the vertex set of G is called semi-regular, if all vertex orbits are of the same size.

A natural combinatorial counterpart to astral configurations can be defined as follows. A cubic bipartite graph is called astral if it admits a semi-regular

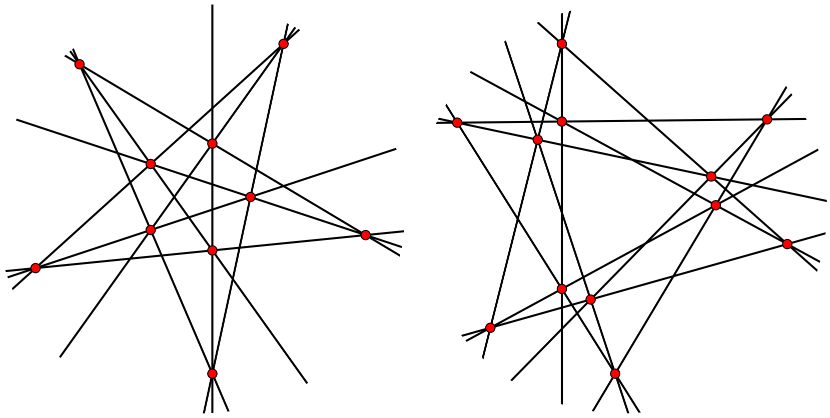

Figure 8: Two astral configurations. The left one has cyclic while the right one has dihedral group of symmetries. Each one is the smallest in its class.

group of automorphism $\Gamma \leq \mathrm{Aut}\, G$ with two black and two white orbits. Using examples of Figure 8 it is clear that there exist cubic bipartite graphs that are cyclically or dihedrally astral. It would be interesting to see what can be said about other groups than can appear in astral cubic bipartite graphs.

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