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EDGE, VERTEX AND MIXED FAULT-DIAMETERS

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Edge, vertex and mixed fault-diameters ⋆

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Abstract

Let $\mathcal{D}^E_q(G)$ denote the diameter of a graph G after deleting any of its q edges, and $\mathcal{D}^V_p(G)$ denote the diameter of G after deleting any of its p vertices. We prove that $\mathcal{D}^E_a(G) \leq \mathcal{D}^V_a(G) + 1$ for all meaningful a. We also define mixed fault diameter $\mathcal{D}^M_{(p,q)}(G)$, where p vertices and q edges are deleted at the same time. We prove that for $0 < l \leq a$, $\mathcal{D}^E_a(G) \leq \mathcal{D}^M_{(a-\ell,\ell)}(G) \leq \mathcal{D}^V_a(G) + 1$, and give some examples.

Key words: (vertex)-connectivity, edge-connectivity, (vertex) fault-diameter, edge fault-diameter, mixed fault-diameter, interconnection network.

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1 Introduction

In the design of large interconnection networks several factors have to be taken into account. A usual constraint is that each processor can be connected to a limited number of other processors and the delays in communication must not be too long. Furthermore, an interconnection network should be fault-tolerant. A lot of work has been done on various aspects of network fault tolerance, see for example the survey [6] and more recent papers [9,13,15]. In particular the fault diameter with faulty vertices which was first studied in [11] and the edge fault-diameter has been determined for many important networks recently [7,8,12,14]. In most papers either only edge faults or only vertex faults are considered, while the case when both edges and vertices may be faulty is studied rarely. For example [9,13] consider Hamiltonian properties assuming a combination of vertex and edge faults. In our recent work on fault-diameter of Cartesian graph products and bundles [2–5], analogous results were found for both fault-diameter and edge fault-diameter. However, the proofs for vertex and edge faults in [2–5] are independent, and our effort to see how results in one case may imply the others was not successful. A natural question remains whether it is possible to design a uniform theory that would enable unified proofs or provide tools to translate results for one type of faults to the other.

It is therefore of interest to study general relationships between invariants under vertex and edge faults. In this paper we define (p,q)-connectivity that generalizes both vertex- and edge-connectivity and (p,q)-mixed fault diameter $\mathcal{D}^M_{(p,q)}(G)$ that generalizes both a-vertex fault-diameter $\mathcal{D}^V_a(G)$ and a-edge fault-diameter $\mathcal{D}^E_a(G)$.

We prove (Theorem 4.7) that for all meaningful values of a and ℓ,

$$\mathcal{D}^E_a(G) \le \mathcal{D}^M_{(a-\ell,\ell)}(G) \le \mathcal{D}^V_a(G) + 1.$$

We also give some examples showing that all bounds are tight.

2 Preliminaries

A *simple graph* $G = (V, E)$ is determined by a *vertex set* $V = V(G)$ and a set $E = E(G)$ of (unordered) pairs of vertices, called the set of *edges*. As usual, we will use the short notation uv for edge $\{u, v\}$. For an edge $e = uv$ we call u and v its *endpoints*. It is convenient to consider union of *elements* of a graph, $S(G) = V(G) \cup E(G)$. Given $X \subseteq S(G)$ then $S(G) \setminus X$ is a subset of elements of G. In particular we can write $X = X_E \cup X_V$, where $X_E \subseteq E(G)$ and $X_V \subseteq V(G)$. Note that in general $S(G) \setminus X$ may not be a set of elements of a graph. As we need notation for subgraphs with some missing (faulty) elements, we will formally define $G \setminus X$, the subgraph of G after deletion of X, as follows:

Definition 2.1 *$G \setminus X$ is the subgraph of $(V(G), E(G) \setminus X_E)$ induced on vertex set $V(G) \setminus X_V$.*

A *walk* between x and y is a sequence of vertices and edges v_0, e_1, v_1, e_2, v_2, $\dots$, v_{k-1}, e_k, v_k where $x = v_0$, $y = v_k$, and $e_i = v_{i-1}v_i$ for each i. A walk with all vertices distinct is called a *path*, and the vertices v_0 and v_k are called the *endpoints* of the path. The *length* of a path P, denoted by $\ell(P)$, is the number of edges in P. The *distance* between vertices x and y, denoted by $\mathrm{d}_G(x, y)$, is the length of a shortest path between x and y in G. If there is not path between x and y we write $\mathrm{d}_G(x, y) = \infty$. The *diameter* of a connected graph G, $\mathrm{d}(G)$, is the maximum distance between any two vertices in G. A path P in G, defined by a sequence $x = v_0, e_1, v_1, e_2, v_2, \dots, v_{k-1}, e_k, v_k = y$ can alternatively be seen as a subgraph of G with $V(P) = \{v_0, v_1, v_2, \dots, v_k\}$ and $E(P) = \{e_1, e_2, \dots, e_k\}$. Note that the reverse sequence gives rise to the same subgraph. Hence we use P for a path either from x to y or from y to x. A graph is *connected* if there is a path between each pair of vertices, and is *disconnected* otherwise. The *connectivity* (or *vertex-connectivity*) of a connected graph G, $\kappa(G)$, is the minimum cardinality over all vertex-separating sets in G. As the complete graph K_n has no vertex-separating sets, we define $\kappa(K_n) = n-1$. We say that G is *k-connected* (or *k-vertex connected*) for any $0 < k \leq \kappa(G)$. The *edge-connectivity* of a connected graph G, $\lambda(G)$, is the minimum cardinality over all edge-separating sets in G. A graph G is said to be *k-edge connected* for any $0 < k \leq \lambda(G)$. In other words: the edge connectivity $\lambda(G)$ of a connected graph G is the smallest number of edges whose removal disconnects G, and the (vertex) connectivity $\kappa(G)$ of a connected graph G (other than a complete graph) is the smallest number of vertices whose removal disconnects G.

It is well known that (see, for example, [1], page 224) $\kappa(G) \leq \lambda(G) \leq \delta_G$, where δ_G is smallest vertex degree of G. Thus if a graph G is k-connected, then it is also k-edge connected. The reverse does not hold in general. Considering the mixed fault-diameters we can also define that G is *(p, q)-connected* if G remains connected after removal of any p vertices and any $q - 1$ edges or of any $p - 1$ vertices and any q edges. Hence (p, q)-connectivity generalizes both vertex- and edge-connectivity. In particular, any graph G is $(\kappa(G), 0)$-connected and $(0, \lambda(G))$-connected. Furthermore, if G is $(k, 0)$-connected, then it is also $(k - i, i)$-connected for any $0 \leq i \leq k$. The proof is straightforward and we leave it to the reader.

3 Edge and vertex fault-diameters

Definition 3.1 *Let G be a k-edge connected graph and $0 \le a < k$. The a-*edge fault-diameter *of G is*

$$\mathcal{D}_a^E(G) = \max\{\mathrm{d}(G \setminus X) \mid X \subseteq E(G), |X| = a\}.$$

Definition 3.2 *Let G be a k-connected graph and $0 \le a < k$. The a-*fault diameter *(or a-*vertex fault-diameter*) of G is*

$$\mathcal{D}_a^V(G) = \max\{\mathrm{d}(G \setminus X) \mid X \subseteq V(G), |X| = a\}.$$

Note that $\mathcal{D}_a^E(G)$ is the largest diameter among diameters of subgraphs of G with a edges deleted, and $\mathcal{D}_a^V(G)$ is the largest diameter over all subgraphs of G with a vertices deleted. In particular, $\mathcal{D}_0^E(G) = \mathcal{D}_0^V(G) = \mathrm{d}(G)$, the diameter of G.

For $a \ge \kappa(G)$, the a-vertex fault-diameter of graph G does not exist, and for $b \ge \lambda(G)$, the b-edge fault-diameter of graph G does not exist. We write $\mathcal{D}_a^E(G) = \infty$, $\mathcal{D}_b^V(G) = \infty$ as some of the graphs are not edge-connected or vertex-connected, respectively.

Remark 3.3 *It is easy to see that for any connected graph G the inequalities below hold.*

(1) $\mathrm{d}(G) = \mathcal{D}_0^E(G) \le \mathcal{D}_1^E(G) \le \mathcal{D}_2^E(G) \le \ldots \le \mathcal{D}_{\lambda(G)-1}^E(G) < \infty.$
(2) $\mathrm{d}(G) = \mathcal{D}_0^V(G) \le \mathcal{D}_1^V(G) \le \mathcal{D}_2^V(G) \le \ldots \le \mathcal{D}_{\kappa(G)-1}^V(G) < \infty.$

In this section we will compare the edge fault-diameter and the vertex fault-diameter with the same number of edges or vertices deleted.

Note that, intuitively, one may expect $\mathcal{D}_a^E(G) \le \mathcal{D}_a^V(G)$ because deleting a vertices in a connected graph always means that at least a edges were deleted. However, this is not the case as the examples below show. From examples it will also follow that the bound of Theorem 3.4 is tight.

Theorem 3.4 *Let G be a k-connected graph and $0 < a < k \le \kappa(G)$. Then*

$$\mathcal{D}_a^E(G) \le \mathcal{D}_a^V(G) + 1.$$

We omit the proof because the result follows from the Theorem 4.7 which will be proved later.

We conclude the section with several examples, including

- graphs with $\mathcal{D}_a^E(G) = \mathcal{D}_a^V(G) + 1$ (examples 3.5, 3.6 and 3.7),
- graphs with $\mathcal{D}_a^E(G) = \mathcal{D}_a^V(G)$ (example 3.8),
- graphs with $\mathcal{D}_a^E(G) < \mathcal{D}_a^V(G)$ (example 3.9).

Example 3.5 *For the cycle $C_n, n \geq 3$ we have $\kappa(C_n) = \lambda(C_n) = 2$, $\mathrm{d}(C_n) = \left\lfloor \frac{n}{2} \right\rfloor$, and $\mathcal{D}_1^E(C_n) = n - 1$, $\mathcal{D}_1^V(C_n) = n - 2$ (for $n = 4$ see Fig. 1).*

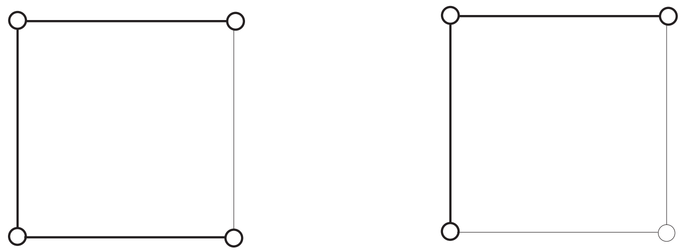

Fig. 1. Cycle C_4 with one faulty link and one faulty node.

Example 3.6 *For the complete graph $K_n, n \geq 3$, clearly $\kappa(K_n) = \lambda(K_n) = n - 1$, $\mathrm{d}(K_n) = 1$, and for each $a \leq n - 2$, $\mathcal{D}_a^E(K_n) = 2$, and $\mathcal{D}_a^V(K_n) = 1$.*

Example 3.7 *For the complete bipartite graph $K_{m,n}$ where $m, n \geq 2$ it is easy to see that $\kappa(G) = \lambda(G) = \min\{m, n\}$, $\mathrm{d}(K_{m,n}) = 2$, and for each $a < \min\{m, n\}$, $\mathcal{D}_a^E(K_{m,n}) = 3$, and $\mathcal{D}_a^V(K_{m,n}) = 2$.*

Example 3.8 *For the hypercube Q_3 we have $\kappa(Q_3) = \lambda(Q_3) = 3$, $\mathrm{d}(Q_3) = \mathcal{D}_1^E(Q_3) = \mathcal{D}_1^V(Q_3) = 3$, and $\mathcal{D}_2^E(Q_3) = \mathcal{D}_2^V(Q_3) = 4$. On Figure 2 four graphs Q_3 with one or two elements missing are given. Cases with maximal diameters are depicted.*

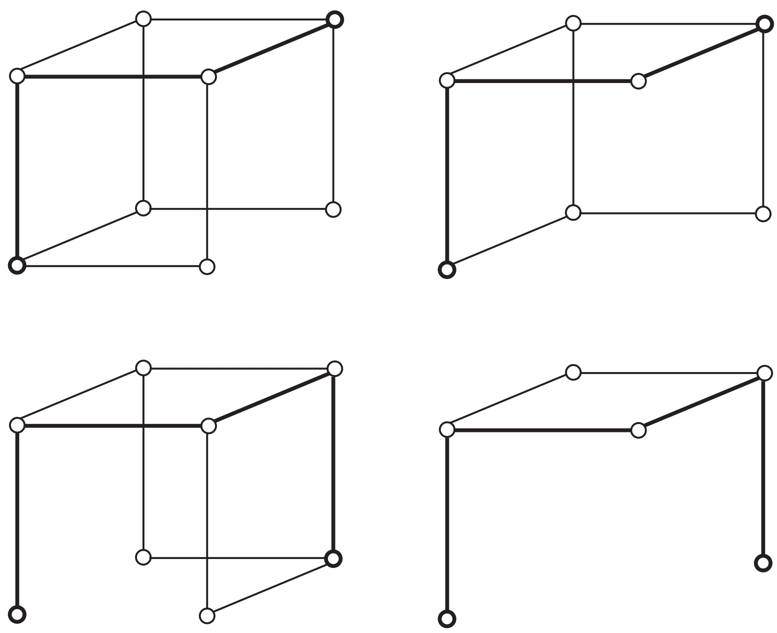

Fig. 2. The cube Q_3 with some elements missing.

Example 3.9 *Let G be a graph obtained from graph $K_2 \Box C_{100}$ where the two vertices of one copy of K_2 are merged into one vertex (see Fig. 3). ($K_2 \Box C_{100}$ is the Cartesian product* [10] of K_2 and C_{100}.) For this graph we have $\kappa(G) = \lambda(G) = 3$, $\mathrm{d}(G) = 50$, $\mathcal{D}_1^E(G) = 51$, and $\mathcal{D}_1^V(G) = 99$. Deleting two edges or two vertices gives diameters $\mathcal{D}_2^E(G) = 99$, $\mathcal{D}_2^V(G) = 100$.

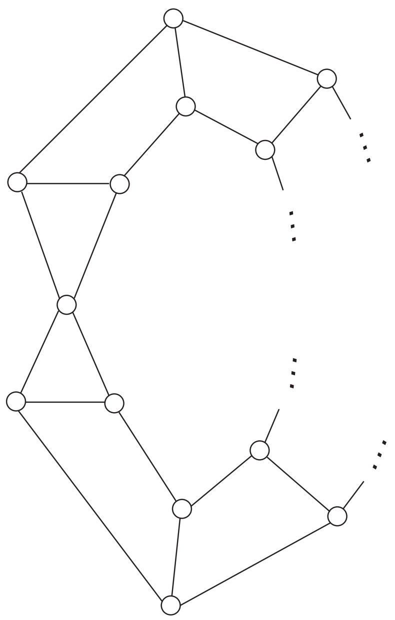

Fig. 3. Graph G from Example 3.9.

4 Mixed fault-diameter

Definition 4.1 *Let G be a (p,q)-connected graph, $(0 \le a < p$ and $0 \le b \le)q$ or $(0 \le a \le p$ and $0 \le b < q)$. The (a,b)-*mixed fault-diameter *of G is*

$$\mathcal{D}^M_{(a,b)}(G) = \max \{\mathrm{d}(G \setminus X) \mid X = X_E \cup X_V,$$

$$X_E \subseteq E(G), X_V \subseteq V(G), |X_V| = a, |X_E| = b\}.$$

Note that by Definition 4.1 the endpoints of edges of set X_E can be in X_V. In this case we actually get a subgraph of G with a vertices and less then b edges deleted, but it is not difficult to see that the diameter of such subgraph is smaller or equal to the diameter of some subgraph of G where exactly a vertices and exactly b edges are deleted. So the condition that the endpoints of edges of set X_E are not in X_V is not necessary to be included in Definition 4.1.

Remark 4.2 *The mixed fault-diameter $\mathcal{D}^M_{(p,q)}(G)$ is the largest diameter among* *diameter of subgraphs of G with q edges and p vertices deleted, hence* $\mathcal{D}^M_{(0,0)}(G) = \mathrm{d}(G)$, $\mathcal{D}^M_{(0,a)}(G) = \mathcal{D}^E_a(G)$ *and* $\mathcal{D}^M_{(a,0)}(G) = \mathcal{D}^V_a(G)$.

Remark 4.3 *Let*

$$\mathcal{H}^V_a = \{G \setminus X \mid X \subseteq V(G), |X| = a\}$$

and

$$\mathcal{H}^E_b = \{G \setminus X \mid X \subseteq E(G), |X| = b\}.$$

It is easy to see that

(1) $\max\{\mathcal{D}^E_b(H) \mid H \in \mathcal{H}^V_a\} = \mathcal{D}^M_{(a,b)}(G)$,
(2) $\max\{\mathcal{D}^V_a(H) \mid H \in \mathcal{H}^E_b\} = \mathcal{D}^M_{(a,b)}(G)$.

Proposition 4.4 *Let G be a (p,q)-connected graph, ($0 \le a < p$ and $0 < b \le q$) or ($0 \le a \le p$ and $0 < b < q$). Then*

$$\mathcal{D}^M_{(a,b)}(G) \le \mathcal{D}^M_{(a+\ell,b-\ell)}(G),$$

where $0 \le \ell \le \min\{b-1, p-a\}$.

Proof. Let G be a (p,q)-connected graph. We will show that in any subgraph of G with a vertices and $b \ne 0$ edges deleted there is a path between any two vertices of length at most $\mathcal{D}^M_{(a+\ell,b-\ell)}(G)$ for any $0 \le \ell \le \min\{b-1, p-a\}$.

Let $X = X_V \cup X_E, X_V \subseteq V(G), X_E \subseteq E(G), |X_V| = a, |X_E| = b \ne 0$. Let $u, v \in G \setminus X$ be two distinct vertices and $0 \le \ell \le \min\{b-1, p-a\}$.

Let $Y' \subset X_E$, where $|Y'| = \ell$, and $uv \notin Y'$. Let $Y_E = X_E \setminus Y'$. Then $|Y_E| = b - \ell$.

For each edge from Y' we choose one endpoint, different from u and v. Let $Y'_V \subseteq V(G)$ be the set of these chosen endpoints. Then $|Y'_V| \le \ell$ as some edges of Y' can have pairwise common endpoints. Note that also some of these vertices can be in X_V. Therefore $|X_V \cup Y'_V| \le a + \ell$. This construction can give many different sets Y'_V. Let Y'_V be arbitrary such set.

Let $Y_V \subset V(G)$ such that $|Y_V| = a + \ell, X_V \cup Y'_V \subseteq Y_V$, and $u, v \notin Y_V$. Let $H = G \setminus (Y_V \cup Y_E)$. H is a subgraph of G with $a + \ell$ vertices and $b - \ell$ edges deleted and $u, v \in H$. As $a + \ell \le p$, $b - \ell \le q$, and ($a + \ell \ne p$ or $b - \ell \ne q$), H is connected. Therefore there is a path P between u and v in H with length $\ell(P) \le \mathcal{D}^M_{(a+\ell,b-\ell)}(G)$. Clearly, H does not contain any vertex from X_V and any edge from X_E. Therefore $H \subseteq G \setminus (X_E \cup X_V)$ and hence $P \subseteq G \setminus (X_E \cup X_V)$. □

Corollary 4.5 *Let G be a k-connected graph, and $0 < a < k$. Then*

$$\mathcal{D}_a^E(G) = \mathcal{D}_{(0,a)}^M(G) \le \mathcal{D}_{(1,a-1)}^M(G) \le \mathcal{D}_{(2,a-2)}^M(G) \le \ldots \le \mathcal{D}_{(a-1,1)}^M(G).$$

Proof. Recall that k-connectivity, i.e. $(k,0)$-connectivity implies $(k-i,i)$-connectivity for $0 \le i \le k$. Apply the theorem. $\Box$

Now we will give an upper bound for the mixed fault-diameter which will give rise to the inequalities involving all tree fault-diameters given in Theorem 4.7.

Proposition 4.6 *Let G be a k-connected graph, $0 < a < k$, $p + q = a$, and $q \neq 0$. Then*

$$\mathcal{D}_{(p,q)}^M(G) \le \mathcal{D}_a^V(G) + 1.$$

Proof. First we delete p vertices in a graph G. As we can do that in $\binom{|V(G)|}{p}$ ways, there are $\binom{|V(G)|}{p}$ different subgraphs of G with p vertices deleted. Let $\mathcal{H} = \mathcal{H}_p^V = \{G \setminus X \mid X \subseteq V(G), |X| = p\}$ be the family of all these subgraphs.

Each subgraph $H \in \mathcal{H}$ is at least $(q+1)$-connected and by Theorem 3.4

$$\mathcal{D}_q^E(H) \le \mathcal{D}_q^V(H) + 1,$$

for each $H \in \mathcal{H}$. Therefore

$$\max\{\mathcal{D}_q^E(H) \mid H \in \mathcal{H}\} \le \max\{\mathcal{D}_q^V(H) \mid H \in \mathcal{H}\} + 1.$$

By Remark 4.3

$$\max\{\mathcal{D}_q^E(H_i) \mid H_i \in \mathcal{H}\} = \mathcal{D}_{(p,q)}^M(G)$$

and

$$\max\{\mathcal{D}_q^V(H_i) \mid H_i \in \mathcal{H}\} = \mathcal{D}_{(p+q,0)}^M(G) = \mathcal{D}_{(a,0)}^M(G) = \mathcal{D}_a^V(G)$$

so the proof is complete. $\Box$

Summarizing the Proposition 4.6 and Corrolary 4.5, we can write the main result of this section

Theorem 4.7 *Let G be a k-connected graph, $0 \le a < k$. Then*

$$\mathcal{D}_a^E(G) = \mathcal{D}_{(0,a)}^M(G) \le \mathcal{D}_{(1,a-1)}^M(G) \le \mathcal{D}_{(2,a-2)}^M(G) \le \ldots \le \mathcal{D}_{(a-1,1)}^M(G) \le \mathcal{D}_a^V(G)+1.$$

We conclude with several examples.

It is easy to see that complete graphs and complete bipartite graphs are examples of graphs with

$$\mathcal{D}_a^E(G) = \mathcal{D}_{(p,q)}^M(G) = \mathcal{D}_a^V(G) + 1.$$

(follows from Examples 3.6 and 3.7).

If for $(a+1)$-connected graph G, $\mathcal{D}_a^E(G) = \mathcal{D}_a^V(G)$, then $\mathcal{D}_{(p,q)}^M(G)$, $p+q=a$, can have the same value as $\mathcal{D}_a^E(G)$ and $\mathcal{D}_a^V(G)$, or is for 1 bigger. For instance, if $G = Q_3$ (see Example 3.8), then $\mathcal{D}_{(1,1)}^M(Q_3) = \mathcal{D}_2^E(Q_3) = \mathcal{D}_2^V(Q_3) = 4$. In Example 4.8, $\mathcal{D}_{(p,q)}^M(G) = \mathcal{D}_a^E(G) + 1 = \mathcal{D}_a^V(G) + 1$.

Example 4.8 *Let W_6 be a wheel graph on six vertices (Fig. 4). We have $\kappa(W_6) = \lambda(W_6) = 3$, $\mathrm{d}(W_6) = \mathcal{D}_1^E(W_6) = \mathcal{D}_1^V(W_6) = 2$, and $\mathcal{D}_2^E(W_6) = \mathcal{D}_2^V(W_6) = 3$, $\mathcal{D}_{(1,1)}^M(W_6) = 4$.*

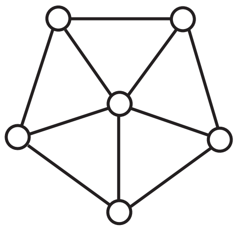

Fig. 4. Graph W_6 from Example 4.8.

There are also graphs where all tree fault-diameters are different.

Example 4.9 *Consider a wheel graph on seven vertices W_7 (Fig. 5). For this graph, $\kappa(W_7) = \lambda(W_7) = 3$, $\mathrm{d}(W_7) = 2$, $\mathcal{D}_2^E(W_7) = 3$, $\mathcal{D}_{(1,1)}^M(W_7) = 5$, $\mathcal{D}_2^V(W_7) = 4$.*

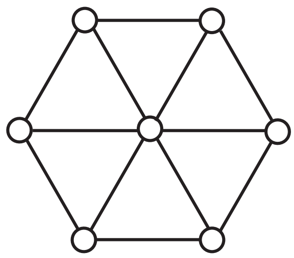

Fig. 5. Graph G from Example 4.9.

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